

DSF preprint 93/52  
SUHEP preprint 582

# Energy-Charge Correlation in the $\pi^+\pi^-\pi^0$ Decay of $K_L$ and of Tagged Neutral Kaons

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## Abstract

We relate the asymmetries in the charged pions energy in the decay into  $\pi^+\pi^-\pi^0$  of  $K_L$  and of the tagged neutral kaons. The former asymmetry is a given combination of  $\Re(\epsilon)$ ,  $\Im(\epsilon)$ , and  $|\epsilon'|$ . Moreover, the non-violating CP asymmetry allows a test for the  $\chi$ PT predictions within the Zel'dovich approach for the final state interaction.

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# 1 Introduction

The kaon system has been the most natural laboratory to study the CP violation since its first evidence in  $K_L \rightarrow \pi\pi$  decays [1].

The phenomenology about CP violation in the  $K \rightarrow \pi\pi$  can be described by the well known parameters  $\epsilon$  and  $\epsilon'$  [2],

$$\epsilon = \tilde{\epsilon} + i \frac{\Im(A_0)}{\Re(A_0)}, \quad (1)$$

$$\epsilon' = \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \frac{\Re(A_2)}{\Re(A_0)} \left( \frac{\Im(A_2)}{\Re(A_2)} - \frac{\Im(A_0)}{\Re(A_0)} \right), \quad (2)$$

$$i A_I \equiv \langle (\pi\pi)_I | H_W | K^0 \rangle, \quad (3)$$

where  $\delta_2$  and  $\delta_0$  are the  $\pi\pi$  phase shifts [3],

$$\delta_2 - \delta_0 + \frac{\pi}{2} = (47 \pm 5)^\circ. \quad (4)$$

The parameter  $\epsilon$  is connected with the CP violation in the mass matrix of the kaons through  $\tilde{\epsilon}$ , which appears in the expressions of the mass eigenstates  $K_L$  and  $K_S$ ,

$$K_S = \frac{(1 + \tilde{\epsilon})K^0 + (1 - \tilde{\epsilon})\bar{K}^0}{\sqrt{2(1 + |\tilde{\epsilon}|^2)}} = \frac{K_1 + \tilde{\epsilon}K_2}{\sqrt{1 + |\tilde{\epsilon}|^2}}, \quad (5)$$

$$K_L = \frac{(1 + \tilde{\epsilon})K^0 - (1 - \tilde{\epsilon})\bar{K}^0}{\sqrt{2(1 + |\tilde{\epsilon}|^2)}} = \frac{K_2 + \tilde{\epsilon}K_1}{\sqrt{1 + |\tilde{\epsilon}|^2}}, \quad (6)$$

where  $K_1$  and  $K_2$  are respectively the CP even and odd eigenstates, and  $\bar{K}^0 = CPK^0$ . On the other side,  $\epsilon'$  is due to the CP violating term in the kaon decay matrix.

The two CP violating parameters  $\epsilon$  and  $\epsilon'$  enter in the following ratios [2]:

$$\eta_{+-} \equiv \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)} \simeq \epsilon + \epsilon', \quad (7)$$

$$\eta_{00} \equiv \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)} \simeq \epsilon - 2\epsilon'. \quad (8)$$

Due to the phenomenological suppression of the  $\Delta I = 3/2$ , the  $\epsilon'$  value is very small ( $\lesssim 10^{-5}$ ) and therefore difficult to be measured experimentally.

The "golden-ratio"

$$\frac{\Gamma[K_S \rightarrow \pi^0 \pi^0] \Gamma[K_L \rightarrow \pi^+ \pi^-]}{\Gamma[K_S \rightarrow \pi^+ \pi^-] \Gamma[K_L \rightarrow \pi^0 \pi^0]} \simeq 1 + 6\Re\left(\frac{\epsilon'}{\epsilon}\right) \quad (9)$$

has been used in measuring  $\Re(\epsilon'/\epsilon)$  by the Collaborations *NA31* at CERN [4] and *E731* at Fermilab [5], which gave the following different results:

$$\begin{array}{ll} (2.0 \pm 0.7) \cdot 10^{-3} & NA\ 31 \\ (0.74 \pm 0.60) \cdot 10^{-3} & E\ 731. \end{array} \quad (10)$$

These measures require more investigations to better understand the real underlying mechanism of the direct CP violation.

The two dedicated experiments *LEAR* [6] and *DAΦNE* [7] were planned to improve these measurements and, at the same time, to enlarge the phenomenology of CP violation in the kaon processes.

The phase of  $\epsilon$ ,  $\phi(\epsilon)$ , is predicted by unitarity to be [3]

$$\phi(\epsilon) \equiv \tan^{-1}\left(\frac{\Im(\epsilon)}{\Re(\epsilon)}\right) = \tan^{-1}\left(\frac{2(M_L - M_S)}{\Gamma_S - \Gamma_L}\right) = (43.68 \pm 0.14)^\circ, \quad (11)$$

very near to the phase of  $\epsilon'$  given by eq. (4). So the result given in eq. (10) indicate that  $|\epsilon'|$  is three order of magnitude smaller than  $|\epsilon|$  and therefore the phase of  $\epsilon$  is, with a good approximation, equal to the phases of  $\eta_{+-}$  and  $\eta_{00}$  measured from the interference of  $K_S$  and  $K_L$  in the decays into two pions. Indeed, in these experiments one finds [3]

$$\phi_{+-} \equiv \tan^{-1}\left(\frac{\Im(\eta_{+-})}{\Re(\eta_{+-})}\right) = (46.6 \pm 1.2)^\circ, \quad (12)$$

$$\phi_{00} \equiv \tan^{-1}\left(\frac{\Im(\eta_{00})}{\Re(\eta_{00})}\right) = (46.6 \pm 2.0)^\circ, \quad (13)$$

in fair agreement one each other and larger but consistent with the value predicted by unitarity.

$\Re(\epsilon)$  may be related to the asymmetry in the semileptonic decays of  $K_L$ , where

$$\frac{\Gamma(K_L \rightarrow \mu^+ \pi^- \nu_\mu) - \Gamma(K_L \rightarrow \mu^- \pi^+ \bar{\nu}_\mu)}{\Gamma(K_L \rightarrow \mu^+ \pi^- \nu_\mu) + \Gamma(K_L \rightarrow \mu^- \pi^+ \bar{\nu}_\mu)} = (0.304 \pm 0.025)\%, \quad (14)$$

$$\frac{\Gamma(K_L \rightarrow e^+ \pi^- \nu_e) - \Gamma(K_L \rightarrow e^- \pi^+ \bar{\nu}_e)}{\Gamma(K_L \rightarrow e^+ \pi^- \nu_e) + \Gamma(K_L \rightarrow e^- \pi^+ \bar{\nu}_e)} = (0.333 \pm 0.014)\%. \quad (15)$$

The two ratios of eqs. (14) and (15), assuming  $\Delta Q = \Delta S$ , are both given by  $2\Re(\epsilon)$ , in quite good agreement with

$$\Re(\epsilon) = (1.637 \pm 0.013) \cdot 10^{-3}, \quad (16)$$

found from  $|\epsilon|$  and  $\phi(\epsilon)$ .

In this paper we give a model independent way to measure a combination of  $\Re(\tilde{\epsilon}) = \Re(\epsilon)$  and  $\Im(\tilde{\epsilon}) = \Im(\epsilon) - \Im(A_0)/\Re(A_0)$  by relating a particular time-dependent integrated CP conserving asymmetry, connecting the strangeness of the neutral kaons to the energy in the Dalitz plot of the charged pions in the decay  $K^0(\bar{K}^0) \rightarrow \pi^+\pi^-\pi^0$ , to the corresponding CP violating term in  $K_L \rightarrow \pi^+\pi^-\pi^0$ .

The time-dependent CP conserving and CP violating asymmetries may be measured in *LEAR* [6] and *DAΦNE* [7] respectively. The measure of the CP conserving asymmetry will provide a test of  $\chi PT$  (Chiral Perturbation Theory) and give information on the final state interactions of the three pions.

## 2 Asymmetries

The amplitude for the decay of  $K_L \rightarrow \pi^+\pi^-\pi^0$  may be given in terms of the corresponding amplitudes into the same channel of the CP eigenstates  $K_1$  and  $K_2$ , which are, up to quadratic order in pion energies and disregarding terms with isospin greater than 3/2 in the weak Hamiltonian [8, 9],

$$A(K_1 \rightarrow \pi^+\pi^-\pi^0) = \tilde{a}e^{i\delta_S(1)} + \tilde{d}\left(\rho^2 + \frac{\Delta^2}{3}\right) + \tilde{b}\rho e^{i\delta_{MS}(1)} + \tilde{e}\left(\rho^2 - \frac{\Delta^2}{3}\right), \quad (17)$$

$$\begin{aligned} &+ i\left[c\Delta e^{i\delta_{MA}(2)} + f\rho\Delta\right], \\ A(K_2 \rightarrow \pi^+\pi^-\pi^0) &= i\left[a e^{i\delta_S(1)} + d\left(\rho^2 + \frac{\Delta^2}{3}\right)\right] + i\left[b\rho e^{i\delta_{MS}(1)} + e\left(\rho^2 - \frac{\Delta^2}{3}\right)\right] \\ &+ \tilde{c}\Delta e^{i\delta_{MA}(2)} + \tilde{f}\rho\Delta, \end{aligned} \quad (18)$$

where

$$\Delta \equiv \frac{E_+ - E_-}{m_K}, \quad \rho \equiv \frac{E_0}{m_K} - \frac{1}{3} \quad (19)$$

are auxiliary variables over the Dalitz plot ( $E_i$  is the energy of  $\pi_i$ ). In the following we will assume SU(2) flavour symmetry, i.e.  $m_{\pi^\pm} = m_{\pi^0}$ . All the coefficients in eqs. (17) and (18) are real in our convention. The CP violating terms are represented by the letters with a tilde;  $\delta_S(1)$  and  $\delta_{MS}(1)$  represent the totally symmetric and mixed symmetric isospin 1 final state interaction, while  $\delta_{MA}(2)$  is connected to the mixed antisymmetric isospin 2 interaction. The factorization for the amplitudes, in terms of the pion phase shifts, relies on the Watson's theorem [10]. The coefficients of the isospin 2 term are due only to the  $\Delta I = 3/2$  operator in the weak interaction, while the other ones are a combination of  $\Delta I = 1/2$  and  $\Delta I = 3/2$ .

In the Standard Model, the direct CP violation in the amplitudes comes from the diagrams corresponding to the "penguin" operators which transform as the  $\Delta I = 1/2$ ,  $|\Delta S| = 1$ ,  $|\Delta Q| = 0$  member of an octet; this suggests to adopt the  $\Im(A_2) = 0$  phase convention as the most natural one. Moreover, it is justified to eliminate the direct phenomenological  $\Delta I = 3/2$  CP violating terms,  $\tilde{c}$  and  $\tilde{f}$ , in the amplitude. By keeping only the CP conserving part of  $A(K_1 \rightarrow \pi^+\pi^-\pi^0)$  one obtains

$$A(K_L \rightarrow \pi^+\pi^-\pi^0) = i \left[ a e^{i\delta_S(1)} + d \left( \rho^2 + \frac{\Delta^2}{3} \right) \right] + i \left[ b \rho e^{i\delta_{MS}(1)} + e \left( \rho^2 - \frac{\Delta^2}{3} \right) \right] + i \tilde{c} \left[ c \Delta e^{i\delta_{MA}(2)} + f \rho \Delta \right], \quad (20)$$

from which we can derive the CP violating asymmetry

$$\Gamma_\Delta(K_L) \equiv \frac{F[\Delta \cdot |A(K_L \rightarrow \pi^+\pi^-\pi^0)|^2] F[1]}{\Gamma[K_L \rightarrow \pi^+\pi^-\pi^0] F[|\Delta|]}. \quad (21)$$

$F[...]$  represents the phase space integration defined in the Appendix B, so that

$$F[|A(K_L \rightarrow \pi^+\pi^-\pi^0)|^2] = \Gamma(K_L \rightarrow \pi^+\pi^-\pi^0). \quad (22)$$

After some algebra we find

$$F \left[ \Delta \cdot |A(K_L \rightarrow \pi^+\pi^-\pi^0)|^2 \right] = 2 [P \Re(\tilde{\epsilon}) + S \Im(\tilde{\epsilon})], \quad (23)$$

where we define

$$F[\Delta A(K_1 \rightarrow \pi^+\pi^-\pi^0)^* A(K_2 \rightarrow \pi^+\pi^-\pi^0)] \equiv P + i S. \quad (24)$$

The complete expression for  $P$  and  $S$  can be found in the Appendix A. Now, eq. (21) becomes

$$\Gamma_{\Delta}(K_L) = \frac{2 [P \Re(\tilde{\epsilon}) + S \Im(\tilde{\epsilon})] F[1]}{\Gamma[K_L \rightarrow \pi^+ \pi^- \pi^0] F[|\Delta|]}. \quad (25)$$

An information on the coefficients of  $\Re(\tilde{\epsilon})$  and  $\Im(\tilde{\epsilon})$  in eq. (25) can be obtained by considering the strangeness-charged pions energy correlation, which may be measured at *LEAR* [6], where it is possible to tag the initial strangeness of the neutral kaons. The latter evolve according to:

$$K^0(t) = \frac{\sqrt{1+|\tilde{\epsilon}|^2}}{\sqrt{2}(1+\tilde{\epsilon})} [K_S e^{-\frac{\Gamma_S t}{2} - iM_S t} + K_L e^{-\frac{\Gamma_L t}{2} - iM_L t}], \quad (26)$$

$$\bar{K}^0(t) = \frac{\sqrt{1+|\tilde{\epsilon}|^2}}{\sqrt{2}(1-\tilde{\epsilon})} [K_S e^{-\frac{\Gamma_S t}{2} - iM_S t} - K_L e^{-\frac{\Gamma_L t}{2} - iM_L t}]. \quad (27)$$

In the previous expressions the standard notation [2] has been adopted.

In order to reach our goal let us consider the CP conserving asymmetry [11]

$$\Sigma_{\Delta}^{+-0}(T) \equiv \int_0^T [\Gamma_{\Delta}(t) - \bar{\Gamma}_{\Delta}(t)] dt, \quad (28)$$

where

$$\Gamma_{\Delta}(t) \equiv F \left[ \Delta \cdot |A(K^0(t) \rightarrow \pi^+ \pi^- \pi^0)|^2 \right], \quad (29)$$

$$\bar{\Gamma}_{\Delta}(t) \equiv F \left[ \Delta \cdot |A(\bar{K}^0(t) \rightarrow \pi^+ \pi^- \pi^0)|^2 \right], \quad (30)$$

and  $\int_0^T \Gamma_{\Delta}(t)(\bar{\Gamma}_{\Delta}(t)) dt$  are reported in the Appendix A.

A straightforward calculation gives the following result [11]:

$$\Sigma_{\Delta}^{+-0}(T) = f(\overrightarrow{F[\Delta A(K_1 \rightarrow \pi^+ \pi^- \pi^0)^* A(K_2 \rightarrow \pi^+ \pi^- \pi^0)]}, T) = P f(\vec{1}, T) + S f(\vec{i}, T), \quad (31)$$

where the  $f(\vec{\mu}, T)$ , reported in two special cases in fig. 1, are defined in the Appendix A. So, by studying the time dependence of  $\Sigma_{\Delta}^{+-0}(T)$  we can extract the coefficients which appear in the expression (25). To get an idea of the size of the effect, in the next section we shall give some theoretical evaluation within the framework of Chiral Perturbation Theory ( $\chi$ PT).

Figure 1: The behaviour of  $f(\vec{1}, T)$  (full line) and  $f(\vec{i}, T)$  (dot line) as a function of  $T$  in units  $\tau_S$ .

### 3 Theoretical expectations

Let us first consider the phase shifts for the final interaction of the pions in the Zel'dovich's approach [9], where the strong interactions are evaluated, in the non relativistic limit, in terms of the s-wave phase shifts for the three pairs of pions that can be formed. So, in the low energy expansion, the relevant phase difference which will appear in our calculations is

$$\delta_S(1) - \delta_{MA}(2) = \frac{a_0}{7} \left[ 13K_{+-} - \frac{K_{+0} + K_{-0}}{2} + \frac{3}{2}(K_{+0} - K_{-0})\frac{\rho}{\Delta} \right], \quad (32)$$

where  $a_0 = 0.2/m_\pi$  is the Weinberg's scattering length and we assume  $a_2 = -2/7 a_0$  [12];  $K_{ij}$  is the momentum of the pion pairs in their center of mass, reported in the Appendix A. We list, for completeness, the expressions for all the phases:

$$\delta_S(1) = \frac{a_0}{7} [13K_{+-} - 2(K_{+0} + K_{-0})], \quad (33)$$

$$\delta_{MS}(1) = \frac{a_0}{7} \left[ 4K_{+-} - \frac{1}{2}(K_{+0} + K_{-0}) - \frac{1}{2}(K_{+0} - K_{-0})\frac{\Delta}{\rho} \right], \quad (34)$$

$$\delta_{MA}(2) = -\frac{3a_0}{7} \left[ \frac{1}{2}(K_{+0} + K_{-0}) + \frac{1}{2}(K_{+0} - K_{-0})\frac{\rho}{\Delta} \right]. \quad (35)$$

The coefficients, involved in the expressions for  $P$  and  $S$ , deduced in the framework of the  $\chi$ PT at order  $p^2$  and  $p^4$  [13] are listed in table I. As it is proved in [13], the values at order  $p^2$  are substantially changed by the corrections at the order  $p^4$  in the energy expansion.

TABLE I

	a	b	c	d	e	f
$O(p^2)$	0.699	-4.55	0.540	-	-	-
$O(p^4)$	0.842	-7.30	0.750	-3.78	-9.46	-0.721

Table 1: Values of the coefficients of eq. (20) at order  $p^2$  and  $p^4$  in units  $10^{-6}$ .

We just considered the moduli given in ref. [13] even though at  $p^4$  in  $\chi$ PT all the coefficients acquire a small imaginary part due to the loop contribution. This is justified as in this paper we prefer to use the approach a' la Zel'dovich, in which all the strong effects are included in the phase shifts.

Now, we can predict the values of the coefficients for both  $\Re(\tilde{\epsilon})$  and  $\Im(\tilde{\epsilon})$  in eq. (25), where only the leading terms in the sin and cos expansion have been retained:

$$\Gamma_{\Delta}(K_L) = 8.4 \cdot 10^{-2} \Re(\tilde{\epsilon}) + 1.6 \cdot 10^{-2} \Im(\tilde{\epsilon}). \quad (36)$$

For  $\Gamma[K_L \rightarrow \pi^+\pi^-\pi^0]$  we assume the experimental value in ref. [3], while for the coefficients  $a, \dots$  we use the order  $p^4$  values in table I, which would give rise to

$$Br(K_L \rightarrow \pi^+\pi^-\pi^0) = 12.4\%, \quad (37)$$

$$Br(K_L \rightarrow \pi^0\pi^0\pi^0) = 21.5\%, \quad (38)$$

$$Br(K_S \rightarrow \pi^+\pi^-\pi^0) = 3.83 \cdot 10^{-5}\%, \quad (39)$$

to be compared with the experimental values

$$Br(K_L \rightarrow \pi^+\pi^-\pi^0) = (12.38 \pm 0.21)\%, \quad (40)$$

$$Br(K_L \rightarrow \pi^0\pi^0\pi^0) = (21.6 \pm 0.8)\%, \quad (41)$$

$$Br(K_S \rightarrow \pi^+\pi^-\pi^0) < 4.9 \cdot 10^{-5}\%. \quad (42)$$

It is wise to stress that, while in all this paper we have not discarded the strong phase shifts corrections and considered the exact Dalitz plot contour, the values showed in the



Figure 2: The contour of the available domain in the Dalitz plot for  $K^0(\overline{K}^0) \rightarrow \pi^+\pi^-\pi^0$ , given by eq. (61) (full line) and in the non-relativistic approximation (dot line).

expressions (37), (38), and (39) have been computed without phases and in the non-relativistic approximation for the final pions, in accordance with the literature. Indeed, adopting the first point of view in computing the same branching ratios, one obtains

$$Br(K_L \rightarrow \pi^+\pi^-\pi^0) = 11.3\%, \quad (43)$$

$$Br(K_L \rightarrow \pi^0\pi^0\pi^0) = 19.5\%, \quad (44)$$

$$Br(K_S \rightarrow \pi^+\pi^-\pi^0) = 3.20 \cdot 10^{-5}\%. \quad (45)$$

This result suggests a more careful analysis in fitting the coefficients for  $k \rightarrow 3\pi$  decays, where one should take in account either the dynamical effect of the strong phase shifts and the exact contour of the Dalitz plot, which is plotted in fig. 2.

By using eq. (36) and the values of  $\Re(\epsilon)$  and  $\Im(\epsilon)$  found in semileptonic decays and the moduli of the  $\eta$  and the value of  $NA31$  [4] for  $\Re(\epsilon'/\epsilon)$  we may separate the contributions for  $\Re(\epsilon)$ ,  $\Im(\epsilon)$  and  $|\epsilon'|$  to  $\Gamma_\Delta(K_L)$ :

$$\Gamma_\Delta(K_L) = 8.4 \cdot 10^{-2} \Re(\epsilon) + 1.6 \cdot 10^{-2} \Im(\epsilon) + 5.0 \cdot 10^{-1} |\epsilon'|. \quad (46)$$

With the  $10^9 K_L$  expected at *DAΦNE* in one year, it will certainly be possible measure a meaningful asymmetry coming from the real part. In order to appreciate the contribution of the imaginary part, and *a fortiori* of  $\epsilon'$  despite the enhancement factor  $\sqrt{2} \frac{\Re(A_0)}{\Re(A_2)} \sim 30$ , a larger number of  $K_L$  is desired (larger luminosity or longer experiment).

In order to achieve a model independent determination of  $P$  and  $S$  one needs to single out in the CP conserving asymmetry  $\Sigma_{\Delta}^{+-0}(T)$  the terms proportional to  $f(\vec{1}, T)$  and  $f(\vec{i}, T)$ . From the expressions given in eqs. (50) and (51) in the Appendix A and the values of the integrals reported in the Appendix B one can realize that, with a few percent approximation,  $P$  and  $S$  are given respectively by

$$P = a \cdot c \cdot F[\Delta^2], \quad (47)$$

$$S = a \cdot c \cdot F[\Delta^2(\delta_S(1) - \delta_{MA}(2))], \quad (48)$$

and so it will be possible to extract the product  $a \cdot c$  from  $P$  and to test the final state interaction of the three pions from the ratio  $S/P^3$ .

Since  $a$  is experimentally well known from  $K_L \rightarrow 3\pi$  decays this seems a good, if not the best, way to get  $c$ , and consequently  $\Gamma(K_S \rightarrow \pi^+\pi^-\pi^0)$ , which is difficult to be measured for the low branching ratio and the necessity to separate it from the contamination of  $K_L$ .

## 4 Conclusions

We have been able to write the CP violating asymmetry in the energy of the charged pions in the  $\pi^+\pi^-\pi^0$  decay of  $K_L$  in terms of two parameters which may be determined by studying the corresponding time-dependent asymmetry in the decays of the neutral kaons with tagged initial strangeness.

With the values given for these parameters by  $\chi PT$  and with the evaluation of the final state interaction given by Zel'dovich one should predict for the asymmetry the value

$$\Gamma_{\Delta}(K_L) = 8.4 \cdot 10^{-2} \Re(\epsilon) + 1.6 \cdot 10^{-2} \Im(\epsilon) + 5.0 \cdot 10^{-1} |\epsilon'|. \quad (49)$$

within reach of the experiment *DAΦNE*, but with a small sensitivity to  $\Im(\epsilon)$  and even smaller to the value of  $\epsilon'$  indicated by *NA31* and *E731* experiments.

The experimental study of the time dependence of the CP conserving asymmetry  $\Sigma_{\Delta}^{+-0}(T)$ , needed to get the coefficients which appear in eq. (49), would supply the determination of

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<sup>3</sup>We are grateful to Prof. H. Leutwyler for bringing this point to our attention.

$\Gamma(K_S \rightarrow \pi^+ \pi^- \pi^0)$ , test the  $\chi PT$  prediction and give information on the final state interaction of the three pions.

### Acknowledgments

It's a great pleasure to thank Prof. H. Leutwyler for clarifying conversations and valuable suggestions and Dr. Gian Piero Mangano for useful discussions and continuous encouragement.

## Appendix A

The fundamental quantities defined by eq. (24), up to order  $p^4$ , are

$$\begin{aligned}
P = & F \left[ a \, c \, \Delta^2 \cos(\delta_S(1) - \delta_{MA}(2)) + b \, c \, \Delta^2 \rho \cos(\delta_{MS}(1) - \delta_{MA}(2)) + \right. \\
& + \left( (d+e) \, c \, \Delta^2 \rho^2 + (d-e) \, c \, \frac{\Delta^4}{3} \right) \cos \delta_{MA}(2) + a \, f \, \Delta^2 \rho \cos \delta_S(1) + \\
& + \left. b \, f \, \Delta^2 \rho^2 \cos \delta_{MS}(1) + (d+e) \, f \, \Delta^2 \rho^3 + (d-e) \, f \, \frac{\Delta^4}{3} \rho \right], \tag{50}
\end{aligned}$$

$$\begin{aligned}
S = & F \left[ a \, c \, \Delta^2 \sin(\delta_S(1) - \delta_{MA}(2)) + b \, c \, \Delta^2 \rho \sin(\delta_{MS}(1) - \delta_{MA}(2)) - \right. \\
& - \left( (d+e) \, c \, \Delta^2 \rho^2 + (d-e) \, c \, \frac{\Delta^4}{3} \right) \sin \delta_{MA}(2) + a \, f \, \Delta^2 \rho \sin \delta_S(1) + \\
& + \left. b \, f \, \Delta^2 \rho^2 \sin \delta_{MS}(1) \right]. \tag{51}
\end{aligned}$$

The other terms do not contribute due to their odd parity in  $\Delta$  and  $\rho$ . The phase space integrals involved in the previous expressions are shown in the next Appendix.

Using eqs. (26), (27), (29), and (30), as in ref. [11], we found

$$\begin{aligned}
\int_0^T \Gamma_\Delta(t)(\bar{\Gamma}_\Delta(t)) dt &= \frac{1}{2[1 + |\tilde{\epsilon}|^2 \pm 2\Re(\tilde{\epsilon})]} \left\{ \frac{1 - e^{-\Gamma_S T}}{\Gamma_S} \left[ \Gamma_\Delta(K_1 \rightarrow \pi^+ \pi^- \pi^0) + \right. \right. \\
&+ \left. |\tilde{\epsilon}|^2 \Gamma_\Delta(K_2 \rightarrow \pi^+ \pi^- \pi^0) + 2 \Re \left( \tilde{\epsilon} F[\Delta \cdot A^*(K_1 \rightarrow \pi^+ \pi^- \pi^0) A(K_2 \rightarrow \pi^+ \pi^- \pi^0)] \right) \right] + \\
&+ \frac{1 - e^{-\Gamma_L T}}{\Gamma_L} \cdot \left[ \Gamma_\Delta(K_2 \rightarrow \pi^+ \pi^- \pi^0) + |\tilde{\epsilon}|^2 \Gamma_\Delta(K_1 \rightarrow \pi^+ \pi^- \pi^0) + \right. \\
&+ \left. 2 \Re \left( \tilde{\epsilon} F[\Delta \cdot A^*(K_2 \rightarrow \pi^+ \pi^- \pi^0) A(K_1 \rightarrow \pi^+ \pi^- \pi^0)] \right) \right] + \\
&\pm \Gamma_\Delta(K_1 \rightarrow \pi^+ \pi^- \pi^0) f(\vec{\tilde{\epsilon}}, T) \pm \Gamma_\Delta(K_2 \rightarrow \pi^+ \pi^- \pi^0) f(\vec{\tilde{\epsilon}}^*, T) + \\
&\pm \left[ f \left( \overrightarrow{F[\Delta \cdot A^*(K_1 \rightarrow \pi^+ \pi^- \pi^0) A(K_2 \rightarrow \pi^+ \pi^- \pi^0)]}, T \right) + \right. \\
&+ \left. |\tilde{\epsilon}|^2 f \left( \overrightarrow{F[\Delta \cdot A(K_1 \rightarrow \pi^+ \pi^- \pi^0) A^*(K_2 \rightarrow \pi^+ \pi^- \pi^0)]}, T \right) \right] \Big\},
\end{aligned} \tag{52}$$

where the upper (lower) sign corresponds to  $K^0$  ( $\bar{K}^0$ ), while once defined  $\vec{\mu} \equiv (\Re(\mu), \Im(\mu))$ , we have [11]

$$f(\vec{\mu}, T) \equiv \frac{2}{\vec{\Gamma}^2} [(1 - \cos \Delta m T e^{\frac{-(\Gamma_S + \Gamma_L)T}{2}}) \vec{\mu} \cdot \vec{\Gamma} + (\vec{\mu} \wedge \vec{\Gamma})_3 \sin \Delta m T e^{\frac{-(\Gamma_S + \Gamma_L)T}{2}}], \tag{53}$$

with:

$$\begin{aligned}
\Delta m &\equiv M_L - M_S, \\
\vec{\Gamma} &\equiv \left( \frac{\Gamma_S + \Gamma_L}{2}, \Delta m \right), \\
(\vec{\mu} \wedge \vec{\Gamma})_3 &= \Re(\mu) \Delta m - \Im(\mu) \frac{\Gamma_S + \Gamma_L}{2}.
\end{aligned} \tag{54}$$

The kinematical factors used in this paper are

$$K_{ij} \equiv \left( \frac{m_\pi}{2} (2Q - 3T_k) \right)^{\frac{1}{2}}, \tag{55}$$

where the  $T_k$  are the kinetic energy of  $\pi_k$  in the kaon rest frame and

$$m_\pi = (m_{\pi^0} + 2m_{\pi^+})/3, \tag{56}$$

$$Q \equiv m_K - m_{\pi^0} - 2m_{\pi^+} = (83.562 \pm 0.032) MeV. \tag{57}$$

## Appendix B

The Dalitz plot variables  $r$  and  $\phi$  are

$$T_0 \equiv \frac{Q}{3}(1 + r \cos \phi), \quad (58)$$

$$T_{\pm} \equiv \frac{Q}{3} \left[ 1 + r \cos \left( \frac{2\pi}{3} \mp \phi \right) \right]. \quad (59)$$

Given a function  $h(r, \phi)$ , we define

$$F(h) \equiv \frac{1}{(4\pi)^3 m_K} \frac{\sqrt{3}}{18} Q^2 \int \int r \, dr \, d\phi \, h(r, \phi) \quad (60)$$

(in particular  $F(1)$  is the phase space factor).

The curve limiting the kinematically allowed region is, in the limit of exact  $SU(2)$  flavour symmetry,

$$1 - (1 + \alpha)r^2 - \alpha r^3 \cos 3\phi = 0, \quad (61)$$

with

$$\alpha = \frac{2Qm_K}{(2m_K - Q)^2}. \quad (62)$$

The values of the integrals used in this paper are

$$\begin{aligned} F[1] &= 1.954 \cdot 10^{-3} MeV, \\ F[|\Delta|] &= 7.694 \cdot 10^{-5} MeV, \\ F[\rho^2] &= 1.404 \cdot 10^{-6} MeV, \\ F[\Delta^2] &= 4.212 \cdot 10^{-6} MeV, \\ F[\rho^3] &= -3.291 \cdot 10^{-9} MeV, \\ F[\rho\Delta^2] &= 9.873 \cdot 10^{-9} MeV, \\ F[\rho^4] &= 2.025 \cdot 10^{-9} MeV, \\ F[\rho^2\Delta^2] &= 2.025 \cdot 10^{-9} MeV, \\ F\left[\frac{\Delta^4}{3}\right] &= 6.076 \cdot 10^{-9} MeV, \end{aligned}$$

$$\begin{aligned}
F\left[\frac{\rho\Delta^4}{3}\right] &= 2.132 \cdot 10^{-11} MeV, \\
F[\rho^3\Delta^2] &= 7.108 \cdot 10^{-12} MeV, \\
F[\Delta^2(\delta_S(1) - \delta_{MA}(2))] &= 7.597 \cdot 10^{-7} MeV, \\
F[\rho\Delta^2\delta_S(1)] &= -3.174 \cdot 10^{-9} MeV, \\
F[\rho\Delta^2(\delta_{MS}(1) - \delta_{MA}(2))] &= -8.995 \cdot 10^{-10} MeV, \\
F[\rho^2\Delta^2\delta_{MS}(1)] &= 8.390 \cdot 10^{-11} MeV, \\
F[\rho^2\Delta^2\delta_{MA}(2)] &= -9.106 \cdot 10^{-11} MeV, \\
F\left[\frac{\Delta^4}{3}\delta_{MA}(2)\right] &= -2.732 \cdot 10^{-10} MeV,
\end{aligned}$$

In order to compute them the contour equation (61) has been numerically solved.

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